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Transport Properties of Partially Ionized Argon in a Magnetic Field

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A renewed interest in the calculation of transport coefficients stems from the need to have reliable data to be used for computational fluid dynamics modeling of high enthalpy flows of aerospace interest. Calculations have been performed for the accurate determination of transport coefficients of partially ionized argon in a magnetic field. The calculations employed the Chapman–Enskog method up to very high orders of approximation to calculate the tensorial transport coefficients of the plasma. The study considers the general case of ionization nonequilibrium in a wide temperature range under the action of an imposed magnetic field. Representative results from this extensive set of calculations are reported and some recommendations for use in computational fluid dynamics modeling are drawn.

Nomenclature

$\mathbf{B}$	=	magnetic induction
$\mathbf{D}_{jk}$	=	diffusion coefficients
$\mathbf{D}_j^T$	=	thermal diffusion coefficients
$\mathbf{E}$	=	electric field
$\mathbf{F}^u$	=	thermal conductivity tensors
h_j	=	component enthalpy per unit mass
$\mathbf{J}_{m_j}$	=	component-mass diffusive flux
$\mathbf{J}_Q$	=	conduction current density
$\mathbf{J}_U$	=	internal energy diffusive flux
$\boldsymbol{j}$	=	total current density
M	=	average molar mass
M_j	=	component molar mass
n	=	number of components
$\mathbf{P}_j^u$	=	pressothermal conductivity coefficients
p	=	pressure
p_j	=	component partial pressure
Q_j	=	component molar charge
T	=	temperature
$\mathbf{U}$	=	unit tensor
$\mathbf{v}$	=	hydrodynamic velocity
$\boldsymbol{x}_j$	=	diffusion driving force
β	=	Hall parameter
δ_{jk}	=	Kronecker delta
η	=	viscosity coefficients
$\boldsymbol{\lambda}_e$	=	electrical conductivity
$\boldsymbol{\lambda}_{ej}^p$	=	pressoelectrical conductivity
$\boldsymbol{\lambda}_e^T$	=	thermoelectrical conductivity
$\boldsymbol{\lambda}'$	=	partial thermal conductivity
$\boldsymbol{\lambda}$	=	true thermal conductivity
$\boldsymbol{\lambda}_T^u$	=	thermal conductivity
$\boldsymbol{\mu}$	=	dynamic viscosity tensor
μ_v	=	bulk viscosity coefficient
π_0	=	normal mean stress
ρ	=	total mass density
ρ_j	=	component-mass density
$\boldsymbol{\tau}$	=	stress tensor
$\boldsymbol{\tau}_0^S$	=	traceless symmetric part of the stress tensor
$(\nabla\mathbf{v})_0^s$	=	traceless symmetric part of velocity gradient

Subscripts

j, k	=	subscripts associated to components
t	=	direction transverse to the magnetic field
$\parallel$	=	direction parallel to the magnetic field
$\perp$	=	direction perpendicular to the magnetic field

I. Introduction

TRANSPORT processes in partially ionized gases moving in a magnetic field have been studied by the Chapman–Enskog method [1] and by Grads 13-moment method [2]. The two approaches are mostly equivalent, but Grads 13-moment method corresponds to the second approximation of the Chapman–Enskog method in terms of Sonine polynomial expansion, so that its accuracy may turn out to be insufficient. Recent interest in magnetohydrodynamics (MHD) effects on hypersonic flows has brought a renewed attention to this field. In a previous paper, theoretical research on the subject has been reviewed [3]. There, only the calculation of the viscosity tensor was carried out for equilibrium argon plasma. In a subsequent work, calculations were extended to all other transport coefficients [4]. In any linear transport theory the diffusion velocities, the heat flux, the stress tensor, the charge, and the current density are linear functions of the spatial gradients of the macroscopic parameters (densities of the species, flow velocity, temperature, pressure) and of the electromagnetic field. The coefficients of these functions are the transport coefficients. The

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Chapman–Enskog method, in particular, derives the transport coefficients from a perturbative treatment of the Boltzmann transport equation. If a magnetic field is present, the transport coefficients must be expressed in a tensorial form. The calculation of the relevant coefficients entails the solution of systems of integro-differential equations. The coefficients are then expanded in a series of orthogonal polynomials (the Sonine polynomials, in fact) and the series truncated at the desired order of approximation. The calculation thus reduces to the solution of systems of linear algebraic equations of order $n \cdot K$, n being the number of species and K the order of approximation of the expansion in Sonine polynomials. As a result, the transport coefficients are expressed as determinants whose coefficients are known functions of the macroscopic parameters, the fields and the usual collision integrals between particles. With regard to the convergence of the preceding expansion, the transport coefficients involving collisions between heavy particles can be assumed to converge at the second approximation; when the electrons make the highest contribution, the convergence is much slower and can be dependent on the temperature and pressure of the plasma: it is the case of the thermal and electrical conductivities and of the electron-heavy particle diffusion coefficients [5–7]. The general formulation has been applied to the calculation of transport properties in nonequilibrium one-temperature argon plasma subject to the magnetic field. The calculations have been carried out for temperatures from 500 to 20,000 K and for values of the electron Hall parameter from 10^{-3} to 100. Several cases have been considered by varying the composition of the plasma from pure argon to completely ionized plasma. The results provide a useful tool to all computational fluid dynamics (CFD) researchers interested in MHD effects. SI units are used throughout.

II. Theory

Transport coefficients have been obtained by the Chapman–Enskog method [1]. The plasma consists of argon atoms, singly charged ions, and electrons. The collision integrals used as physical input data are discussed in detail in our previous work [7]. All reported results are obtained in the fifth approximation in Sonine polynomials. In this respect, the error due to convergence is everywhere smaller than the quoted uncertainty of the collision integrals [7]. The following transport coefficients have been considered: 1) diffusion coefficients, 2) thermal diffusion coefficients, 3) thermal conductivity, 4) electrical conductivity, 5) electrothermal conductivity, 6) pressoelectrical conductivity, and 7) viscosity.

The definition of these coefficients is briefly recalled next. For a more detailed discussion of the theoretical framework, refer to the paper by Giordano [8].

Choose a coordinate system where the x axis is directed along the magnetic field. The transport coefficients have therefore three independent components (except for shear viscosity, which has five). In terms of these coefficients, the flux vectors of mass, electric charge, and energy are written as follows:

1) Mass diffusion fluxes

$$\mathbf{J}_{m_\mathbf{j}} = \frac{\rho}{p}\frac{M}{M}\sum_{k=1}^{n}\frac{M_k}{M}\mathbf{D}_{jk}\cdot \boldsymbol{x}_k - \frac{\mathbf{1}}{T}\mathbf{D}_j^T\cdot\nabla T \qquad j,k = 1,\dots,n \quad (1)$$

where

$$\boldsymbol{x}_j = \nabla p_j - \frac{Q_j}{M_j}\rho_j(\mathbf{E}+\mathbf{v}\times\mathbf{B}) \qquad j,k=1,\dots,n \quad (2)$$

The molar electric charges Q_j are defined by Eq. (3) of [8]. The diffusion and thermal diffusion tensors are

$$\mathbf{D}_{jk} = \begin{pmatrix} (\mathbf{D}_{jk})'' & 0 & 0 \\ 0 & (\mathbf{D}_{jk})^{\perp} & -(\mathbf{D}_{jk})^{t} \\ 0 & (\mathbf{D}_{jk})^{t} & (\mathbf{D}_{jk})^{\perp} \end{pmatrix} \quad (3)$$

$$\mathbf{D}_j^T = \begin{pmatrix} (\mathbf{D}_j^T)'' & 0 & 0 \\ 0 & (\mathbf{D}_j^T)^{\perp} & -(\mathbf{D}_j^T)^{t} \\ 0 & (\mathbf{D}_j^T)^{t} & (\mathbf{D}_j^T)^{\perp} \end{pmatrix} \quad (4)$$

2) Conduction current density

$$\mathbf{J}_Q = \sum_{j=1}^{n}\boldsymbol{\lambda}_{e\boldsymbol{j}}^{p}\cdot\nabla p_j + \boldsymbol{\lambda}_e\cdot(\mathbf{E}+\mathbf{v}\times\mathbf{B})\boldsymbol{\lambda}_e^T\cdot\nabla T \quad (5)$$

where

$$\boldsymbol{\lambda}_{e\boldsymbol{j}}^{p} = \frac{\rho}{p}\frac{M_j}{M^2}\sum_{k=1}^{n}Q_k\mathbf{D}_{kj} \quad (6)$$

$$\boldsymbol{\lambda}_e = -\frac{\rho}{pM^2}\sum_{j,k=1}^{n}Q_j\mathbf{D}_{jk}\rho_k Q_k \quad (7)$$

$$\boldsymbol{\lambda}_e^T = -\frac{1}{T}\sum_{j=1}^{n}\frac{Q_j}{M_j}\mathbf{D}_j^T \quad (8)$$

3) Heat flux

$$\mathbf{J}_U = \sum_{j=1}^{n}h_j\mathbf{J}_{m_\mathbf{j}} - \sum_{j=1}^{n}\frac{1}{\rho_j}\tilde{\mathbf{D}}_j^T\cdot\boldsymbol{x}_j - \boldsymbol{\lambda}'\cdot\nabla T = \sum_{j=1}^{n}\mathbf{P}_j^u\cdot\nabla p_j + \mathbf{F}^u\cdot(\mathbf{E}+\mathbf{v}\times\mathbf{B}) - \boldsymbol{\lambda}_T^u\cdot\nabla T \quad (9)$$

where

$$\boldsymbol{\lambda}' = \begin{pmatrix} (\boldsymbol{\lambda}')'' & 0 & 0 \\ 0 & (\boldsymbol{\lambda}')^{\perp} & -(\boldsymbol{\lambda}')^{t} \\ 0 & (\boldsymbol{\lambda}')^{t} & (\boldsymbol{\lambda}')^{\perp} \end{pmatrix} \quad (10)$$

$$\boldsymbol{\lambda}_T^u = \boldsymbol{\lambda}' + \frac{1}{T}\sum_{j=1}^{n}h_j\mathbf{D}_j^T \quad (11)$$

$$\mathbf{P}_j^u = \sum_{k=1}^{n}\left[-\delta_{jk}\frac{1}{\rho_k}\tilde{\mathbf{D}}_k^T + \frac{\rho}{p}\frac{M_jM_k}{M^2}h_k\mathbf{D}_{kj}\right] \quad (12)$$

$$\mathbf{F}^u = \sum_{j=1}^{n}\left[\frac{Q_j}{M_j}\tilde{\mathbf{D}}_j^T - \frac{\rho}{p}h_j\frac{M_j}{M^2}\sum_{k=1}^{n}\rho_kQ_k\mathbf{D}_{jk}\right] \quad (13)$$

4) Stress tensor

The stress tensor is given by

$$\boldsymbol{\tau} = \pi_0\mathbf{U} + \boldsymbol{\tau}_0^S \quad (14)$$

where

$$\pi_0 = -p + \mu_v\nabla\cdot\mathbf{v} \quad (15)$$

$$\boldsymbol{\tau}_0^S = 2\boldsymbol{\mu}\cdot(\nabla\mathbf{v})_0^S \quad (16)$$

The bulk viscosity vanishes when the internal structure of particles is neglected.

Table 1 summarizes the linear dependence of the components of the traceless symmetric stress tensor on those of the traceless symmetric part of the velocity gradient [9]:

Table 1 Relation between the traceless symmetric parts of the stress tensor and the velocity gradient [9]

	$[(\nabla \mathbf{v})_0^S]_{xx}$	$[(\nabla \mathbf{v})_0^S]_{yy}$	$[(\nabla \mathbf{v})_0^S]_{zz}$	$[(\nabla \mathbf{v})_0^S]_{yz}$	$[(\nabla \mathbf{v})_0^S]_{zx}$	$[(\nabla \mathbf{v})_0^S]_{xy}$
$[\boldsymbol{\tau}_0^S]_{xx}$	$2\eta_1$	0	0	0	0	0
$[\boldsymbol{\tau}_0^S]_{yy}$	0	$2\eta_2$	$2(\eta_1 - \eta_2)$	$2\eta_4$	0	0
$[\boldsymbol{\tau}_0^S]_{zz}$	0	$2(\eta_1 - \eta_2)$	$2\eta_2$	$-2\eta_4$	0	0
$[\boldsymbol{\tau}_0^S]_{yz}$	0	$-2\eta_4$	η_4	$-2\eta_1 + 4\eta_2$	0	0
$[\boldsymbol{\tau}_0^S]_{zx}$	0	0	0	0	$2\eta_3$	$2\eta_5$
$[\boldsymbol{\tau}_0^S]_{xy}$	0	0	0	0	$-2\eta_5$	$2\eta_3$

A complete computational scheme is therefore available for the calculation of transport coefficients of partially ionized argon under the action of the magnetic field. The tensorial transport coefficients can be calculated to any desired level of approximation, provided the required collision integrals are available. Also, ionization nonequilibrium can be taken into account in a straightforward manner. However, we recall that this formalism is applicable only when thermal equilibrium can be assumed, so that, in particular, the electron and heavy particle average kinetic energies are equal.

III. Results

An extensive set of calculations has been carried out varying the values for the parameters that control the state of the plasma. The pressure is considered fixed at 1 atm. The temperature is varied in the range of 500–20,000 K, keeping fixed the magnetic field at $B = 1$ T, while the dependence on the magnetic field strength is studied tuning the B value in the range of $10^{-2} - 100$ T, for different values of the temperature ($T = 5000$, 10,000, 15,000, 20,000 K). Concerning the composition, several cases have been considered. Each set of calculations considers the plasma composition fixed while the pressure, the temperature, and the magnetic field strength are varied. These cases analyze a plasma with strong nonequilibrium composition at different ionization degrees. The following cases have been considered: 1) pure argon, 2) $x_{\mathrm{Ar}} = 0.9999$, $x_{\mathrm{Ar}^+} = x_e = 5.0 \times 10^{-5}$, 3) $x_{\mathrm{Ar}} = 0.001$, $x_{\mathrm{Ar}^+} = x_e = 0.4995$, and 4) completely ionized gas.

The argon plasma considered in this work consists of the following components: Ar, Ar^+, e. This assumption is reasonable until the temperature is below 20,000 K (for the equilibrium plasma). Latin subscript indices always refer to the component and run from 1 to 3 in this case [1) Ar, 2) Ar^+, 3) e].

It is to be remembered that, due to the anisotropy introduced by the magnetic field, the transport coefficients have a tensorial nature. Here, all the independent tensor components are reported in a frame of reference with one axis parallel to the magnetic field. The definition of these coefficients in the expression of the fluxes (of mass, charge, momentum, energy) is given in Sec. II.

Selected results are presented for the transport coefficients of argon plasma. More details and the complete set of tabulated results can be found in [10]. Here, in particular, the role of the plasma composition and of the magnetic field strength are analyzed. To this end, two cases are considered: a weakly ionized plasma (case a) and a fully ionized plasma (case b). The former condition relates to the composition of the plasma at low and intermediate temperatures ($T \sim 5000$ K), mainly affected by neutral argon collisions, whereas the latter applies to high-temperature plasma and its properties are controlled by Coulomb interactions.

Results are discussed as far as the following coefficients are concerned: true thermal conductivity, electrical conductivity, and viscosity. Also, an analysis is performed on the main contributions to conduction current density and heat flux.

A. True Thermal Conductivity

In Fig. 1, the parallel component of the true thermal conductivity is shown as a function of temperature for different compositions of the plasma. It is seen that the thermal conductivity increases strongly with the ionization degree. Also, the temperature dependence is much stronger for the Coulomb interactions (fully ionized case) than for collisions involving argon atoms (weakly ionized case $\equiv x_{\mathrm{Ar}} = 0.99999$). The equilibrium ionization is larger than 5×10^{-6} for $T > 5000$ K.

1. *Effect of Magnetic Field*

To show the effect of the magnetic field, in Fig. 2, the transverse component of the true thermal conductivity is plotted for the two cases as a function of temperature at $p = 1$ atm and $B = 1$ T. Obviously, it is the electron component that is affected by the magnetic field at these field strengths. Therefore, the transverse component is nonnegligible only if the ionization degree is substantial. We note also that the electron-argon collision cross

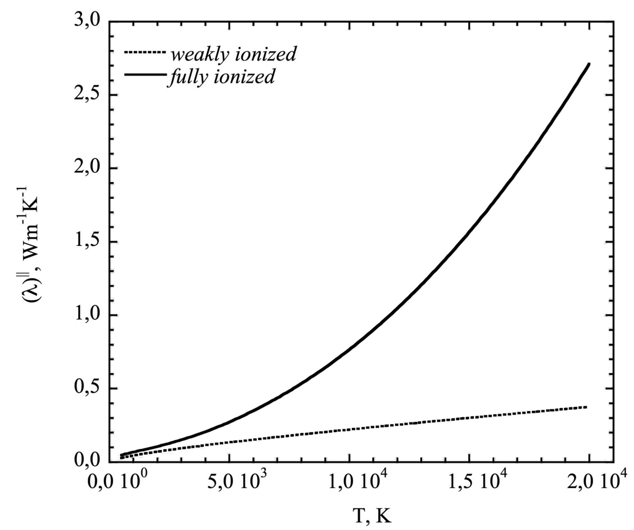


Fig. 1 Parallel component of the true thermal conductivity as a function of temperature at $p = 1$ atm, $B = 1$ T.

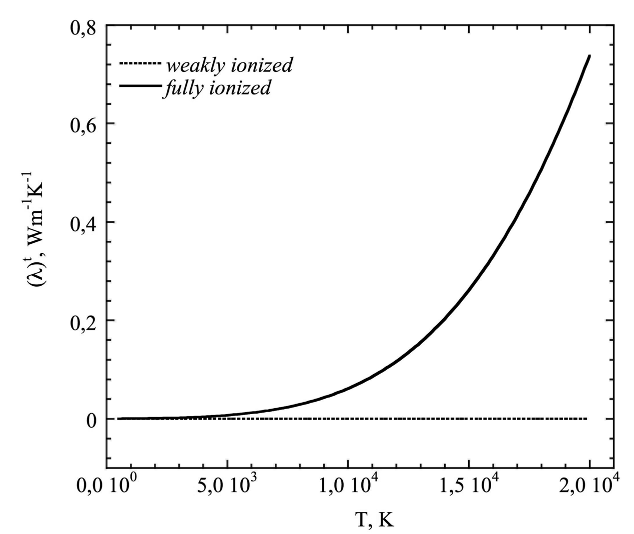


Fig. 2 Transverse component of the true thermal conductivity as a function of temperature at $p = 1$ atm, $B = 1$ T.

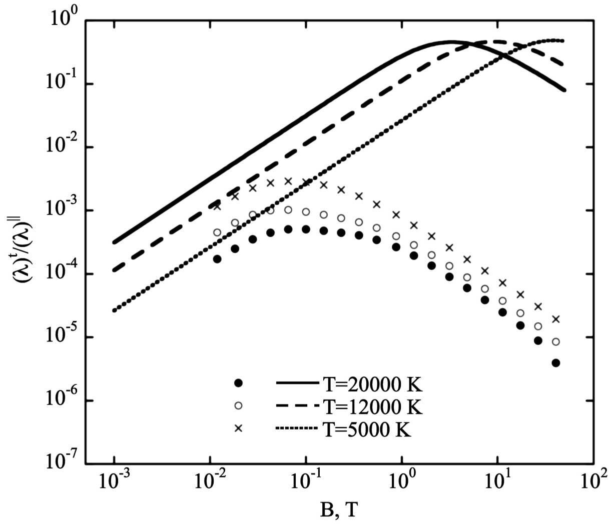


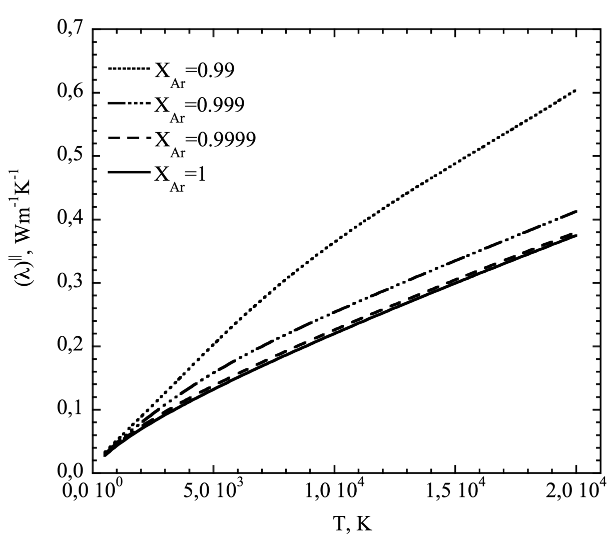
Fig. 3 Ratio of transverse to parallel components of the true thermal conductivity as a function of magnetic field at $p = 1$ atm for different temperature values (full lines: fully ionized; dashed lines: weakly ionized).

sections are smaller than the Coulomb cross sections at the same temperature, so that, for a given magnetic field strength, the electron Hall parameter is larger and the effect of the magnetic field correspondingly enhanced when these collisions contribute to transport.

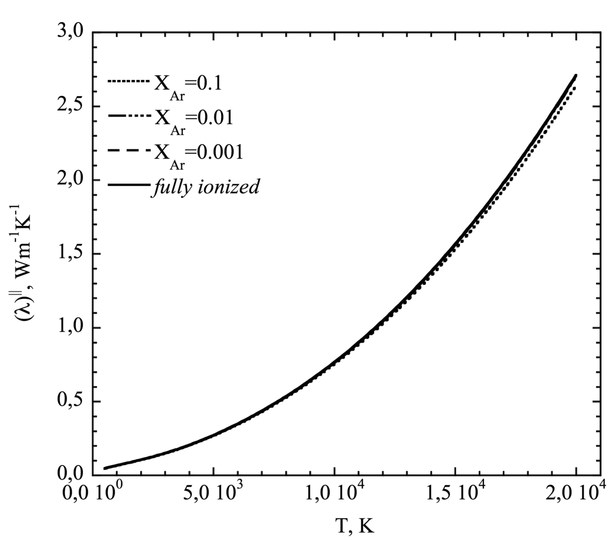
This is best seen in Fig. 3 where the ratio of transverse to parallel components of the true thermal conductivity as a function of magnetic field is reported for the two cases. For weakly ionized plasma, the Hall parameter does not significantly change with temperature and the maximum effect ($\beta_e \sim 1$) is attained for $B \sim 0.1$ T. The transverse component stays very much smaller than the parallel one for this case. On the contrary, in the fully ionized case, the cross sections are larger, and decrease strongly with increasing temperature. The curves are shifted to higher B values but, because the electron component is large enough, the effect in absolute values is much stronger in this case.

2. *Sensitivity*

The sensitivity of the results to composition changes is shown in Figs. 4a and 4b. As mentioned before, the thermal conductivity is strongly affected by the presence of the electron component. Therefore, the limiting result of pure argon is achieved when the



a)



b)

Fig. 4 Sensitivity of the parallel component of the true thermal conductivity to composition changes: a) deviations from pure argon, b) deviations from fully ionized argon.

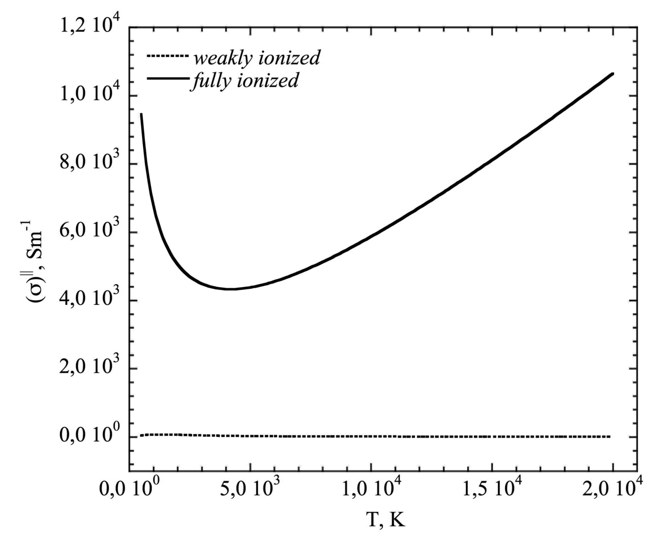


Fig. 5 Parallel component of the electrical conductivity as a function of temperature at $p = 1$ atm, $B = 1$ T (weakly ionized case $\equiv x_{Ar} = 0.99999$).

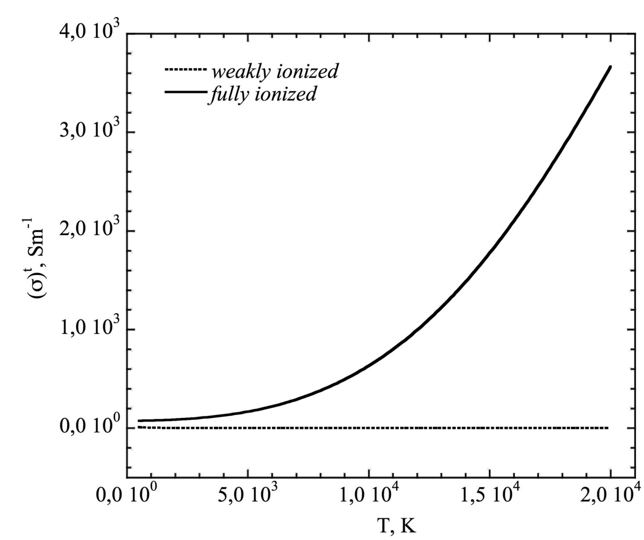


Fig. 6 Transverse component of the electrical conductivity as a function of temperature at $p = 1$ atm, $B = 1$ T (weakly ionized case $\equiv x_{Ar} = 0.99999$.

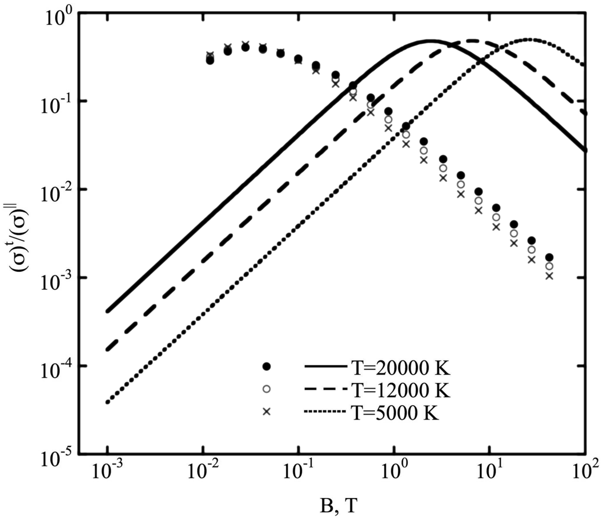
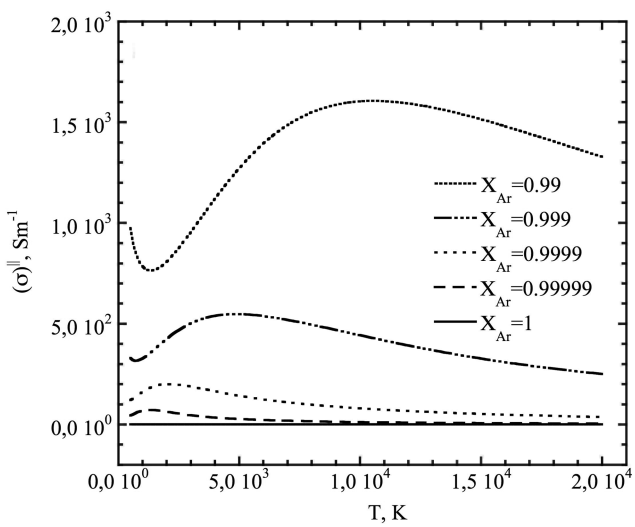
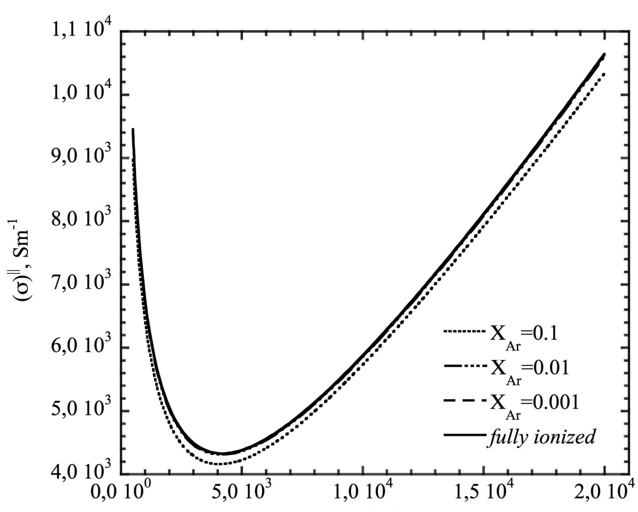


Fig. 7 Ratio of transverse to parallel components of the electrical conductivity as a function of magnetic field at $p = 1$ atm, for different temperature values (lines: fully ionized; symbols: weakly ionized).



a)



b)

Fig. 8 Sensitivity of the parallel component of the electrical conductivity to composition changes: a) deviations from pure argon, b) deviations from fully ionized argon.

ionization degree is lower than 5×10^{-4} (Fig. 4a). By the same token, the results are not very much affected by a small concentration of neutral argon. Figure 4b shows that the thermal conductivity departs from the fully ionized gas behavior only when the argon molar fraction exceeds several percent.

B. Electrical Conductivity

The electrical conductivity is essentially due to electron diffusion. Therefore, nonnegligible values are attained only if the electron component is present. Figure 5 shows the electrical conductivity in the fully ionized case. In comparison, the weakly ionized gas has a negligible conductivity.

1. *Effect of Magnetic Field*

The magnetic field affects essentially only the electron component. Therefore, we shall expect a much bigger effect in this case, relative to the parallel component, where the absolute value depends almost entirely on the electron transport. Figure 6 shows the absolute values of the transverse components of the electrical conductivity for the two cases. It reproduces the behavior of Fig. 2 and the same considerations apply. In this case, however, the effect of the magnetic field is relatively stronger as can be appreciated in Fig. 7. Even at low ionization degrees, the transverse component is comparable to the parallel component.

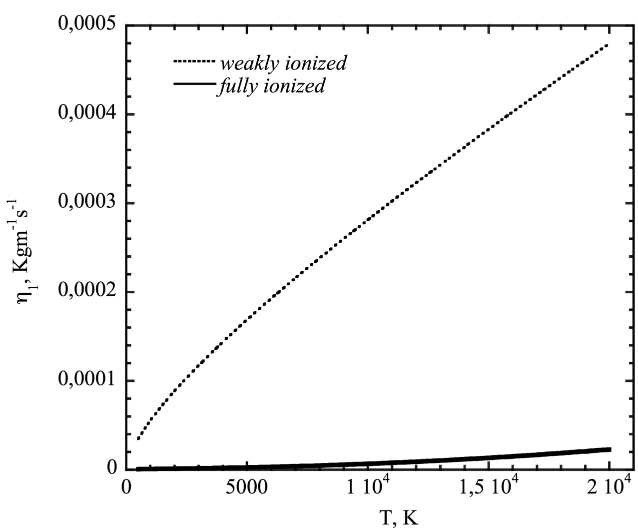


Fig. 9 Viscosity coefficient η_1 as a function of temperature at $p = 1$ atm, $B = 1$ T (weakly ionized case $\equiv x_{Ar} = 0.99999$).

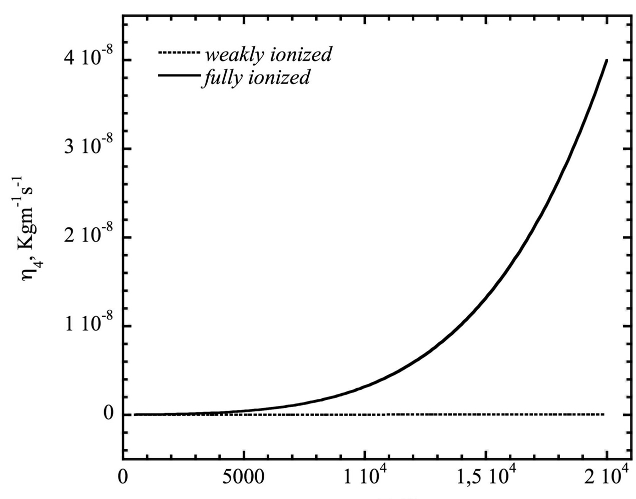


Fig. 10 Viscosity coefficient η_4 as a function of temperature at $p = 1$ atm, $B = 1$ T (weakly ionized case $\equiv x_{Ar} = 0.99999$).

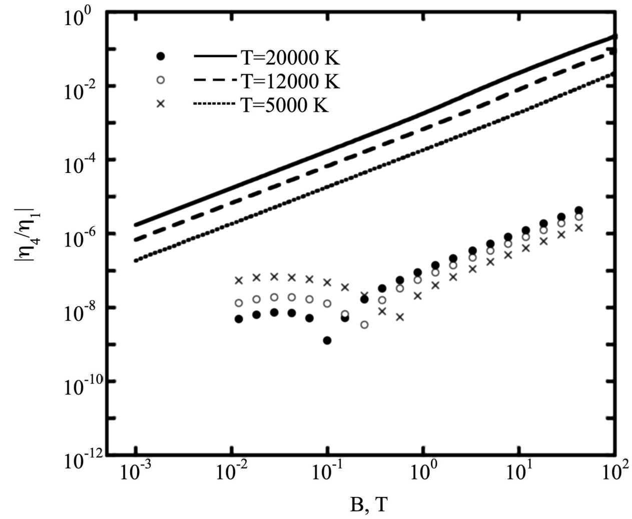
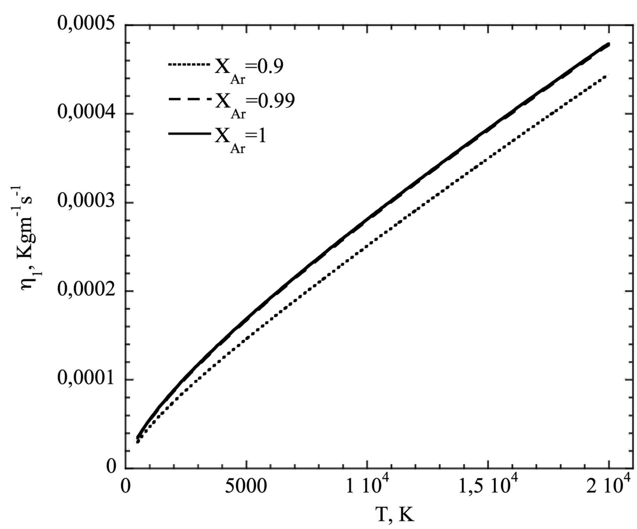
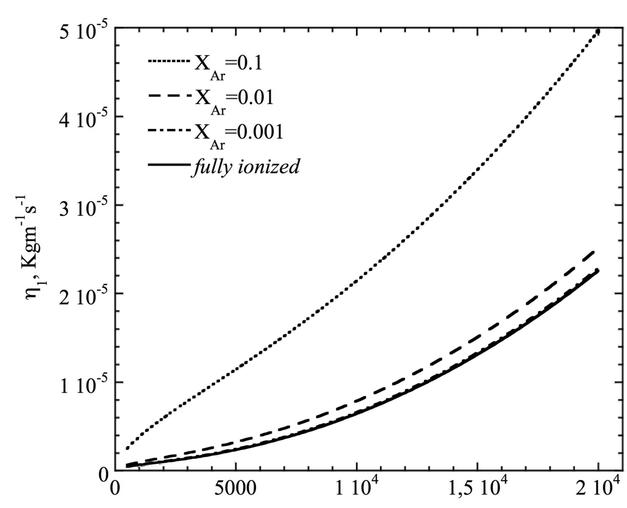


Fig. 11 Ratio of η_4 and η_1 viscosity coefficients as a function of magnetic field at $p = 1$ atm for different temperature values (lines: fully ionized; symbols: weakly ionized).



a)



b)

Fig. 12 Sensitivity of η_1 viscosity coefficient to composition changes: a) deviations from pure argon, b) deviations from fully ionized argon.

2. Sensitivity

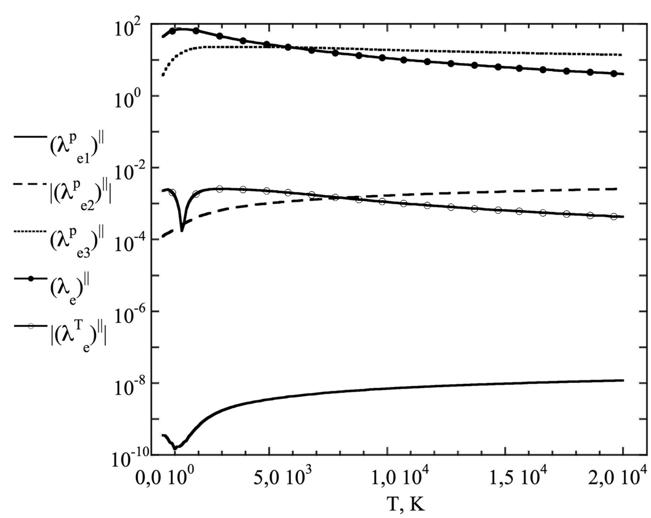
The electrical conductivity grows fast with the ionization degree. Figure 8a shows that, even at $\alpha \sim 5 \times 10^{-4}$, the electrical conductivity is comparable to that of the fully ionized gas. Analogously, Fig. 8b shows that a small percentage of argon does not affect the results as long as this fraction is below 10%.

C. Viscosity

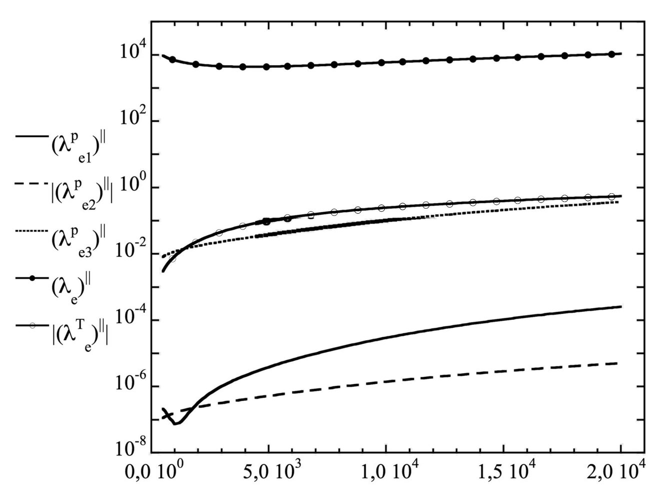
The viscosity, which describes the transport of linear momentum, is controlled by the collisions between heavy particles. We therefore expect that its value decreases with the ionization degree, because the electrons do not contribute and because the Coulomb cross sections are larger, and that the viscosity coefficients are not affected much by the presence of the magnetic field, except at very strong fields. The first point is shown in Fig. 9, which reports the behavior of the component of the viscosity tensor not affected by the field η_1 as a function of the temperature for the two considered cases.

1. Effect of Magnetic Field

As mentioned in the preceding section, being determined by the heavy components, viscosity is hardly affected by the field, except at very high fields. At $B = 1$ T the components of the viscosity tensor that depend on the presence of the magnetic field are very small. Figure 10 shows η_4 as an example. The absolute values are almost

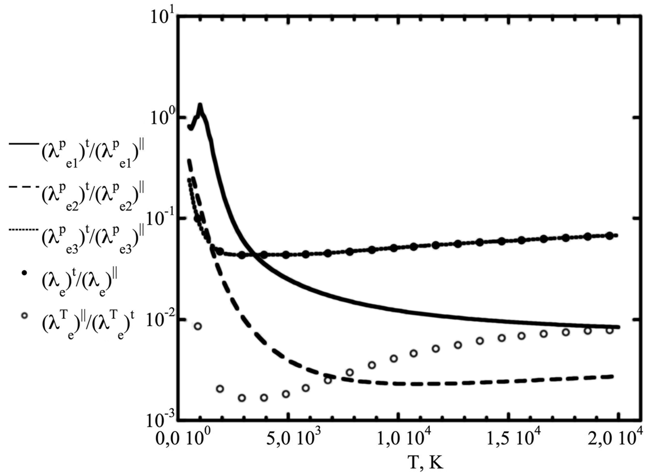


a)

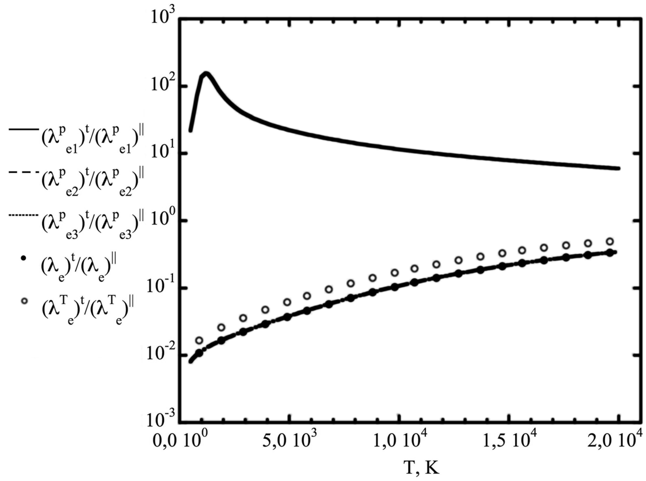


b)

Fig. 13 Parallel component of pressoelectrical, electrical, and thermoelectrical conductivity as a function of temperature at $p = 1$ atm and $B = 1$ T: a) weakly ionized plasma, b) fully ionized plasma.



a)



b)

Fig. 14 Ratio of transverse to parallel components of pressoelectrical, electrical, and thermoelectrical conductivity as a function of temperature at $p = 1$ atm and $B = 1$ T: a) weakly ionized plasma, b) fully ionized plasma.

four orders of magnitude lower than for η_1. Figure 11 shows the behavior of the η_4 to η_1 ratio as a function of the magnetic field. Because the governing parameter here is the ion Hall parameter, which stays much smaller than one, except for very high fields, this ratio tends to increase monotonously as the magnetic field strength increases. Also, the effect is very limited, in particular for low ionization degrees where the viscosity is controlled by the neutral gas, not affected by the field.

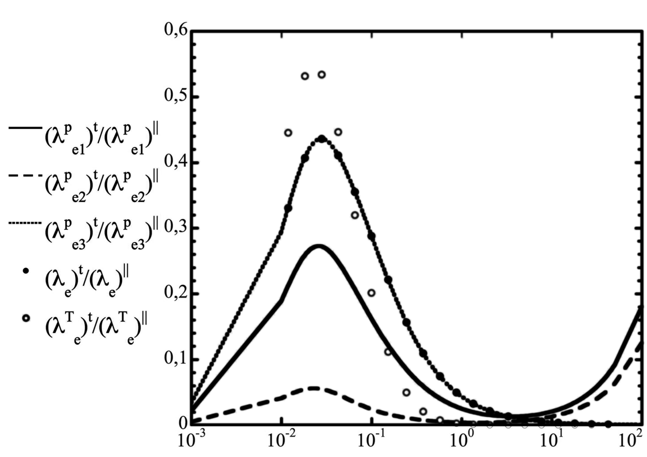
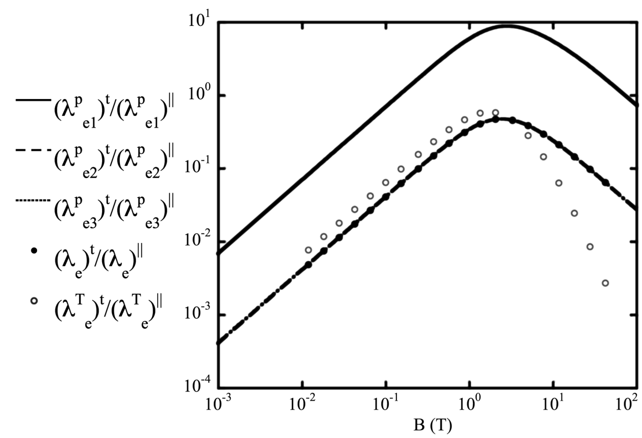


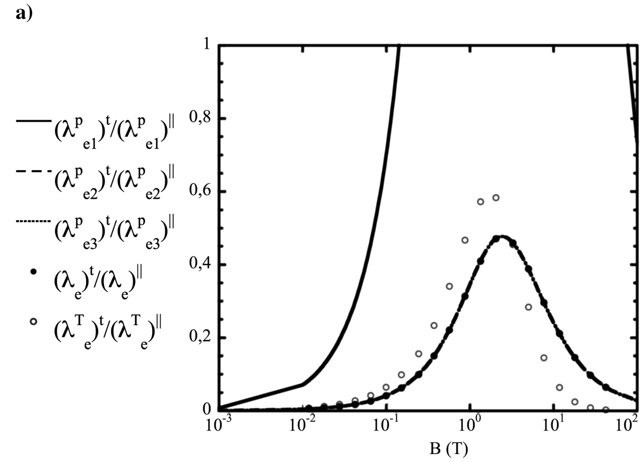
Fig. 15 Ratio of transverse to parallel components of pressoelectrical, electrical, and thermoelectrical conductivity as a function of magnetic field at $p = 1$ atm and $T = 5000$ K (weakly ionized plasma $x_{Ar} = 0.99999$).

2. Sensitivity

Viscosity is dominated by neutral argon. It therefore approaches the pure argon result when ionization is smaller than 5% (Fig. 12a). This also means that a small percentage of neutral argon is enough to increase the viscosity coefficient with respect to the fully ionized



a)



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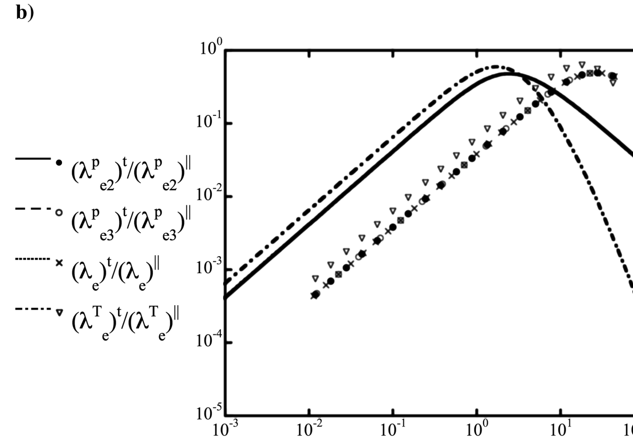


c)

Fig. 16 Ratio of transverse to parallel components of pressoelectrical, electrical, and thermoelectrical conductivity as a function of magnetic field at $p = 1$ atm and $T = 20000$ K: (a) logarithmic scale, b) linear scale, c) fully ionized plasma (symbols: $T = 5000$ K, lines: $T = 20,000$ K).

limit. Figure 12b shows that 1% of argon is sufficient to change the viscosity coefficient considerably.

D. Conduction Current Density

We turn now to a discussion of the different contributions to the conduction current density, Eq. (5), which we rewrite here for convenience:

$$\mathbf{J}_Q = \sum_{j=1}^{n} \boldsymbol{\lambda}_{ej}^{p} \cdot \nabla p_j + \boldsymbol{\lambda}_e \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \boldsymbol{\lambda}_e^{T} \cdot \nabla T \tag{17}$$

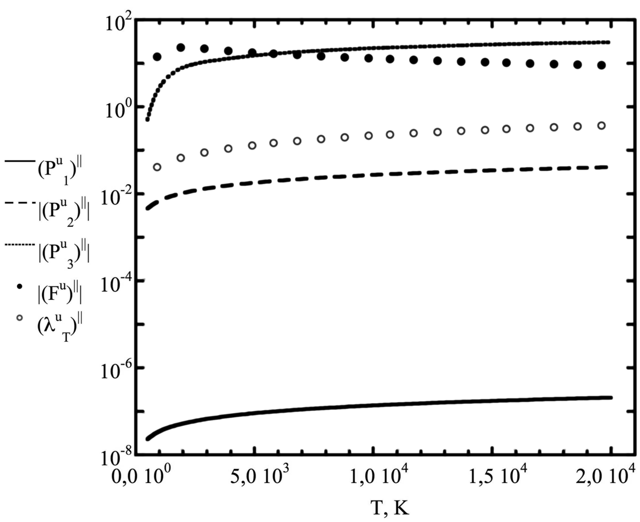
It is apparent from that expression that, in principle, the current is not determined solely by the Ohm contribution (second term). In Fig. 13a, the parallel component of the coefficients that enter the expression in Eq. (17) are plotted as a function of the temperature for a weakly ionized gas at $p = 1$ atm. Some of the coefficients can be negative. In this case the main contributions come from $\boldsymbol{\lambda}_e$ and $\boldsymbol{\lambda}_{e3}^{p}$. At relatively high temperatures, the latter is even larger than the former, so that only an estimation of the driving forces (gradients, fields) in Eq. (17) can give an indication of which are the major effects that control the current. About four orders of magnitude lower are the $\boldsymbol{\lambda}_{e2}^{p}$ and $\boldsymbol{\lambda}_e^{T}$ terms. Finally, completely negligible appears to be the $\boldsymbol{\lambda}_{e1}^{p}$ coefficient.

For fully ionized plasma, the situation is different, as can be appreciated from Fig. 13b. Here only the $\boldsymbol{\lambda}_e$ contribution is important, whereas $\boldsymbol{\lambda}_{e3}^{p}$ is of the same order of the electrothermal conductivity $\boldsymbol{\lambda}_e^{T}$.

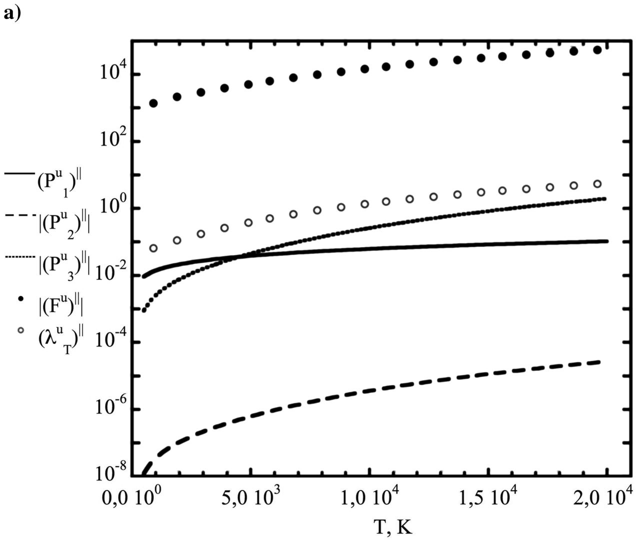
In a real flowfield, where the composition can change drastically from pure argon to substantial ionization, at least the following terms must be taken into account: $\boldsymbol{\lambda}_{e3}^{p}$ for weak ionization, $\boldsymbol{\lambda}_{e3}^{p}$ and $\boldsymbol{\lambda}_e$ for moderate ionization, and $\boldsymbol{\lambda}_e$ for strong ionization; whereas, the most important correction factors come from $\boldsymbol{\lambda}_{e2}^{p}$ for weak ionization, $\boldsymbol{\lambda}_{e2}^{p}$ and $\boldsymbol{\lambda}_e^{T}$ for moderate ionization, and $\boldsymbol{\lambda}_{e3}^{p}$ and $\boldsymbol{\lambda}_e^{T}$ for strong ionization.

For weakly ionized gas, at $B = 1$ T, the effect of the magnetic field is strong, is the same on both main contributions, and is temperature-independent. For fully ionized gas, the effect grows with temperature as the electron Hall parameter grows and the transverse component becomes of the same order as the parallel component for temperatures bigger than 10,000 K (Fig. 14).

In the following, we analyze the behavior of the principal contributions as a function of the magnetic field strength. At a small ionization degree, the coefficients depend weakly on temperature, as it can be appreciated in Fig. 15, where the ratio of the transverse to the parallel component of the different coefficients is reported for a weakly ionized plasma at $p = 1$ atm and $T = 5000$ K. The effect is substantial for all components even at relatively low field strengths. For fields on the order of 1 T, the Hall parameter is large enough so that the orthogonal components, together with the transverse components, tend to vanish, and the plasma behaves as an inviscid

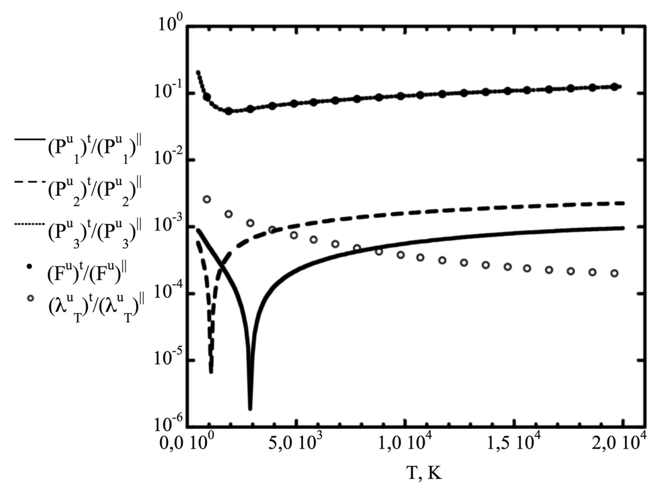


a)

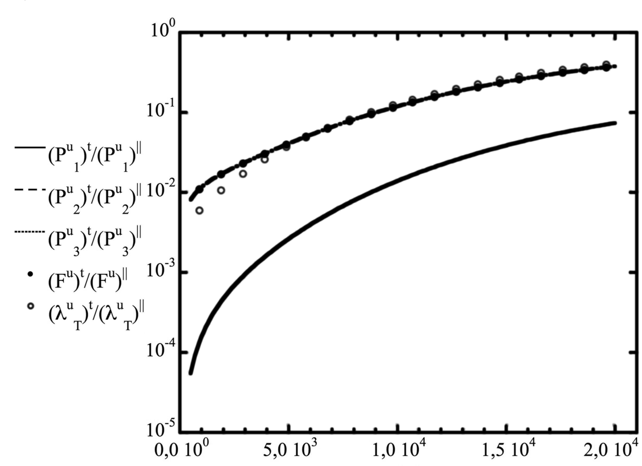


b)

Fig. 17 Parallel component of pressothermal conductivity, electrothermal conductivity, and thermal conductivity as a function of the temperature for $p = 1$ atm and $B = 1$ T: a) weakly ionized gas, b) fully ionized gas.



a)



b)

Fig. 18 Ratio of transverse to parallel components of pressothermal conductivity, electrothermal conductivity, and thermal conductivity as a function of the temperature for $p = 1$ atm and $B = 1$ T, a) weakly ionized plasma, b) fully ionized plasma.

fluid in the direction normal to the magnetic field. Analogous results are obtained for the fully ionized plasma (Figs. 16b and 16c), except that the curves are shifted to stronger fields as expected. The $\boldsymbol{\lambda}_{e1}^{p}$ shows a peculiar behavior, but this has to be compared with the negligible absolute values of this coefficient. In this case, there is a more marked temperature dependence of the coefficient and of the Hall parameter, as noted several times (Fig. 16c).

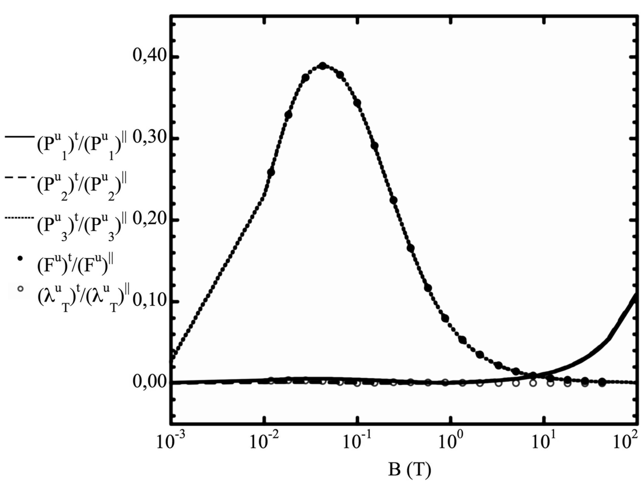
E. Heat Flux

We turn now to a discussion of the different contributions to the heat flux, Eq. (9):

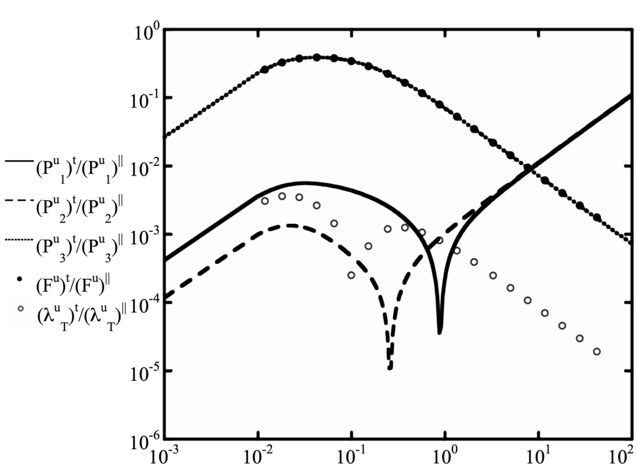
$$\mathbf{J}_U = \sum_{j=1}^{n} h_j \mathbf{J}_{m_j} - \sum_{j=1}^{n} \frac{1}{\rho_j} \tilde{\mathbf{D}}_j^T \cdot \boldsymbol{x}_j - \boldsymbol{\lambda}' \cdot \nabla T = \sum_{j=1}^{n} \mathbf{P}_j^u \cdot \nabla p_j + \mathbf{F}^u \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \boldsymbol{\lambda}_T^u \cdot \nabla T \tag{18}$$

It is apparent from the expression [Eq. (18)] that, in principle, the flux is not determined solely by the Fourier contribution (third term). In Fig. 17a, the parallel component of the coefficients that enter the expression Eq. (18) are plotted as a function of the temperature for a $p = 1$ atm weakly ionized gas. Some of the coefficients can be negative.

For weakly ionized gases (Fig. 17a), the main contribution comes from $\mathbf{F}^u$ and $\mathbf{P}_3^u$.



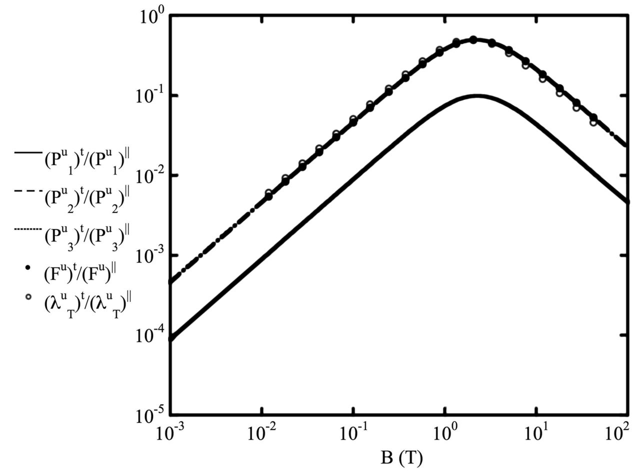
a)



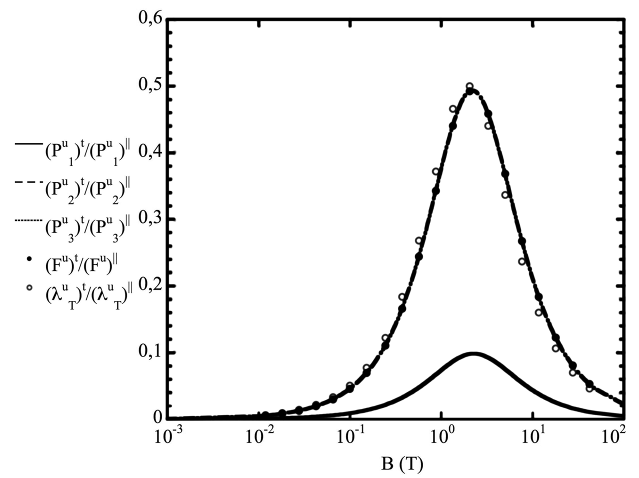
b)

Fig. 19 Ratio of transverse to parallel components of pressothermal conductivity, electrothermal conductivity, and thermal conductivity as a function of magnetic field at $p = 1$ atm, $T = 5000$ K: a) linear scale, b) logarithmic scale (weakly ionized plasma $x_{\mathrm{Ar}} = 0.99999$).

For fully ionized gas (Fig. 17b), instead, the main factor is $\mathbf{F}^u$. Again, we want to stress that, in a real flowfield, these estimations must be complemented with an estimation of the forces that drive the transport phenomena. This analysis, however, can be a guideline for the selection of useful approximations. Inspection of Figs. 17a and



a)



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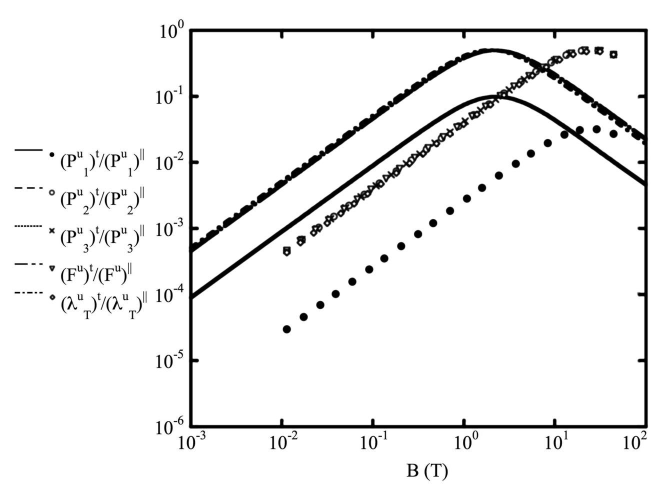

c)

Fig. 20 Ratio of transverse to parallel components of pressothermal conductivity, electrothermal conductivity, and thermal conductivity as a function of magnetic field at $p = 1$ atm, $T = 20000$ K:a) logarithmic scale. (b) linear scale, c) fully ionized plasma (symbols: $T = 5000$ K, lines: $T = 20,000$ K).

17b shows that at least $\mathbf{P}_3^u$, $\mathbf{F}^u$, and $\boldsymbol{\lambda}_T^u$ must be considered for the calculation of the heat flux in a wide temperature range.

As already noted several times, the magnetic field has the strongest effect on the electron component. For the present case, this results in strong anisotropy effects, manifesting in $\mathbf{P}_3^u$ and $\mathbf{F}^u$. This is shown in Fig. 18a for the weakly ionized gas case. For this composition, the latter are also the most relevant contributions to the heat flux.

In the fully ionized limit, all coefficients, except for $\mathbf{P}_1^u$ whose value is largely ininfluent, follow the same trend and show a marked temperature dependence (Fig. 18b), varying from a few percent at low temperature to a few tens of percent at high temperature.

As the magnetic field is varied, the coefficients that depend on electron transport show an increase in anisotropy. This effect goes through a maximum when the electron Hall parameter is on the order of one, and then decreases. For a weakly ionized gas, only $\mathbf{P}_3^u$ and $\mathbf{F}^u$ show this effect (Fig. 19a), whereas the other coefficients depend on heavy particle transport and are not affected by the field, except for very high fields (Fig. 19b). On the contrary, in the fully ionized plasma, the electrons control the transport of thermal energy so that all coefficients display the same behavior as a function of the magnetic field (Figs. 20a and 20b). As usual, the effect sets in at higher values of the magnetic field, as compared to the weakly ionized gas case. The temperature dependence of these curves is again described in terms of the temperature dependence of the electron Hall parameter (Fig. 20c).

IV. Conclusions

Extensive calculations have been performed for the accurate determination of transport coefficients of partially ionized argon in a magnetic field. The calculations employed the Chapman–Enskog method up to very high orders of approximation to estimate the tensorial transport coefficients of the plasma. The study considers the general case of ionization nonequilibrium in a wide temperature range under the action of an imposed magnetic field. Reported results show that anisotropy, induced by the presence of the magnetic field, can strongly alter the transport coefficients, thus making the isotropy assumption untenable. This is particularly relevant at low pressures and at high temperatures, i.e., regimes of interest for hypersonic applications, where even moderate magnetic fields of technological interest can produce deviations from the isotropic behavior on the order of tens of percent.

Acknowledgment

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References

[1] Ferziger, J. H., and Kaper, H. G., *Mathematical Theory of Transport Processes in Gases*, North-Holland, Amsterdam, 1972.

[2] Zhdanov, V. M., *Transport Phenomena in a Multicomponent Plasma* (in Russian), Energoizdat, Moscow, 1982.

[3] Bruno, D., Capitelli, M., and Dangola, A., "Transport Coefficients of Partially Ionized Gases: A Revisitation," AIAA Paper 2003-4039, June 2003.

[4] Bruno, D., Laricchiuta, A., Capitelli, M., Catalfamo, C., Chickaoui, A., Kustova, E. V., and Giordano, D., "Transport Properties of Equilibrium Argon Plasma in a Magnetic Field," AIAA Paper 2004-2161, June 2004.

[5] Devoto, R. S., "Transport Properties of Ionized Monatomic Gases," *Physics of Fluids*, Vol. 9, No. 6, 1966, pp. 1230–1240. doi:10.1063/1.1761825

[6] Capitelli, M., "Transport Properties of Partially Ionized Plasmas," *Journal de Physique Colloque*, Vol. 38, No. C3, Aug. 1977, pp. 227–237. doi:10.1051/jphyscol:1977325

[7] Bruno, D., Catalfamo, C., Laricchiuta, A., Giordano, D., and Capitelli, M., "Convergence of Chapman–Enskog Calculation of Transport Coefficients of Magnetized Argon Plasma," *Physics of Plasmas*, Vol. 13, No. 7, 2006, p. 072307. doi:10.1063/1.2221675.

[8] Giordano, D., "Hypersonic-Flow Governing Equations with Electromagnetic Fields," AIAA Paper 2002-2165, June 2002.

[9] de Groot, S. R., and Mazur, P., *Non-Equilibrium Thermodynamics*, Dover, New York, 1984.

[10] Bruno, D., Capitelli, M., Catalfamo, C., Laricchiuta, A., Kustova, E. V., and Chikhaoui, A., "Transport Coefficients of Argon Plasma with Electromagnetic Fields" (to be published), ESA, STR-254, Noordwijk, The Netherlands.